Dynamics of Walls: Fermion Scattering in Magnetic Field

L. Campanelli[1,2,*], P. Cea[1,2,†], G. L. Fogli[1,2,‡] and L. Tedesco[1,2,§]

[1]*Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy*

[2]*INFN - Sezione di Bari, I-70126 Bari, Italy*

November, 2001

Abstract

We investigate the scattering of fermions off walls in the presence of a magnetic field. We consider both the bubble wall and the kink domain wall. By solving the Dirac equation for fermions in the presence of a domain wall in an external magnetic field, we investigate the dependence on the magnetic field of the transmission and reflection coefficients. In the case of kink domain wall, we also consider the solutions localized on the wall. The possibile role of the ferromagnetic domain walls in the dynamics of the early Universe is also discussed.

PACS number(s): 11.15.Ha

[*]Electronic address: `leonardo.campanelli@ba.infn.it`

[†]Electronic address: `Cea@ba.infn.it`

[‡]Electronic address: `Fogli@ba.infn.it`

[§]Electronic address: `luigi.tedesco@ba.infn.it`

I. INTRODUCTION

Recently, a considerable amount of renewed interest has emerged in the physics of topological defects produced during cosmological phase transitions [1]. It is known since long time that, even in a perfectly homogeneous continuous phase transition, defects will form if the transition proceeds sufficiently faster than the relaxation time of the order parameter [2–5]. In such a non-equilibrium transition, the low temperature phase starts to form, due to quantum fluctuations, simultaneously and independently in many parts of the system. Subsequently, these regions grow together to form the broken-symmetry phase. When different causally disconnected regions meet, the order parameter does not generally match and a domain structure is formed.
In the standard electroweak phase transition the neutral Higgs field is the order parameter which is expected to undergo a continuum phase transition. Actually, if we compare the lower bound recently established by the ALEPH Collaboration [6],

$$M_H \; > \; 107.7 \, \text{GeV} \; \text{at} \; 95 \, \% \; \text{C.L.} \, , \tag{1.1}$$

with the results of non-perturbative lattice simulations [7], we are induced to safely exclude a first order electroweak phase transition [8]. However, a first order phase transition can be nevertheless obtained with an extension of the Higgs sector of the Standard Model, or in the Minimal Supersymmetric Standard Model. If we assume that this is the case, it can be also conjectured that the observed baryon asymmetry may be generated at the primordial electroweak phase transition [9–11].
In the case in which the phase transition is induced by the Higgs sector of the Standard Model, the defects are domain walls across which the field flips from one minimum to the other. The defect density is then related to the domain size and the dynamics of the domain walls is governed by the surface tension σ. The existence of the domain walls, however, is still questionable: it was pointed out by Zel'dovich, Kobazarev and Okun [12] that the gravitational effects of just one such wall stretched across the universe would introduce a large anisotropy into the relic blackbody radiation. For this reason the existence of such walls was excluded. Quite recently, however, it has been suggested [13, 14] that the effective surface tension of the domain walls can be made vanishingly small due to a peculiar magnetic condensation induced by fermion zero modes localized on the wall. As a consequence, the domain wall acquires a non zero magnetic field perpendicular to the wall, and it becomes almost invisible as far as the gravitational effects are concerned. In a similar way, even for the bubble walls it has been suggested [15] that strong magnetic fields may be produced as a consequence of non vanishing spatial gradients of the classical value of the Higgs field. Thus, we are led to suppose that in general the magnetic field vanishes in the regions where the scalar condensate is constant: it can be different from zero only in the regions where the scalar condensate varies, i.e. in a region of the order of the wall thickness.
It is worthwhile to stress that in the realistic case where the domain wall interacts with the plasma, the magnetic field penetrates into the plasma over a distance of the order of the penetration length, which at the epoch of the electroweak phase transition is about an order of magnitude greater than the wall thickness. This means that fermions which scatter on the wall feel an almost constant magnetic field over a spatial region much greater than the wall thickness. So that we can assume that the magnetic field is constant.
The aim of this paper is to study the scattering of fermions off the walls; in particular,

within the so-called *defect mediated electroweak baryogenesis*. Since the dynamical generation of the baryon asymmetry at the electroweak transition is related to the transmission and reflection coefficients, it is interesting to see how the magnetic field localized at the wall modifies these coefficients. In a previous paper [16] we considered Dirac fermions with momentum asymptotically perpendicular to the wall surface; this corresponds to neglect the motion parallel to the wall surface. In the present paper we include the effects due to the motion of fermions parallel to the wall. In particular, we will see that, in the scattering of fermions off domain walls in the presence of a constant magnetic field, there are localized states corresponding to fermions which asymptotically have positive energy. These localized states are a peculiar characteristic of domain walls and are expected to play an important role in the dynamics of the walls.

The plan of the paper is as follow. In Section 2 we discuss the Dirac equation in the presence of a planar domain wall with a constant magnetic field perpendicular to the wall. We evaluate the reflection and transmission coefficients and compare them with the known results without the magnetic field. Section 3 is devoted to the study of the localized states on the planar domain wall. Finally, some concluding remarks are presented in Section 4. For completeness, in the Appendix we discuss the case of bubble walls in the presence of magnetic field perpendicular to the wall.

II.   KINK DOMAIN WALL

In this Section we consider domain walls which are thought to be formed in a continuous phase transition by the Kibble mechanism [2–5]. When the scalar field develops a non vanishing vacuum expectation value $\langle \phi \rangle = \pm v$, one may assume that there are regions with $\langle \phi \rangle = +v$ and $\langle \phi \rangle = -v$. It is easy to see that the classical equation of motion of the scalar field admits the solution describing the transition layer between two adjacent regions with different values of $\langle \phi \rangle$:

$$\varphi(z) = v \, \tanh\left(\frac{z}{\Delta}\right), \tag{2.1}$$

where Δ is the domain wall thickness, which in the following we will set to 1. We are interested in the scattering of Dirac fermions off planar domain wall in the presence of the electromagnetic field A_μ. Thus, assuming that fermions are coupled to the scalar field through a Yukawa interaction with coupling g_Y, the Dirac equation reads:

$$(i \, \gamma^\mu \partial_\mu - g_Y \, \varphi - e \, \gamma^\mu A_\mu) \, \Psi(x, y, z, t) = 0, \tag{2.2}$$

where e is the electric charge. Putting

$$\xi = g_Y \, v \quad , \quad g(z) = \tanh z, \tag{2.3}$$

where ξ is the fermion mass in the broken phase, Eq. (2.2) becomes

$$(i \, \gamma^\mu \partial_\mu - \xi \, g(z) - e \, \gamma^\mu A_\mu) \, \Psi(x, y, z, t) = 0. \tag{2.4}$$

In order to solve Eq. (2.4), we write

$$\Psi(x, y, z, t) = (i \, \gamma^\mu \partial_\mu + \xi \, g(z) - e \, \gamma^\mu A_\mu) \, \Phi(x, y, z, t), \tag{2.5}$$

2

and insert Eq. (2.5) into Eq. (2.4), using the standard representation [17] for the Dirac matrices. We obtain:

$$\left[-\partial^2 + i\,\xi\gamma^3\partial_z\,g(z) - 2i\,e\,A^\mu\partial_\mu - i\,e\partial^\mu\,A_\mu - \frac{e}{2}\,\sigma^{\mu\nu}\,F_{\mu\nu} + \right.$$
$$\left. -\xi^2\,g^2(z) + e^2\,A_\mu A^\mu \right]\Phi(x,y,z,t) = 0. \tag{2.6}$$

Now, writing

$$\Phi(x,y,z,t) = \chi(x,y,z)\,e^{-i\,E_r\,t}\,, \tag{2.7}$$

where

$$E_r = \begin{cases} +E & \text{for} \quad r = 1 \quad \text{(positive energy solutions)} \\ -E & \text{for} \quad r = 2 \quad \text{(negative energy solutions)} \end{cases}, \tag{2.8}$$

we get:

$$\left[E^2 + \partial_k^2 - 2\,i\,e\,A^k\partial_k + e^2\,A_k A^k - \xi^2 g^2(z) + \right.$$
$$\left. + i\,\xi\gamma^3\partial_z g(z) - \frac{e}{2}\,\sigma^{\mu\nu}F_{\mu\nu} \right]\chi(x,y,z)\,e^{-i\,E_r\,t} = 0\,. \tag{2.9}$$

According to our previous discussion, we can safely assume that A_μ corresponds to a constant magnetic field directed along z with strength $F_{21} = B$. We can then choose the gauge such that:

$$A_\mu = (0, 0, -Bx, 0)\,. \tag{2.10}$$

Setting now:

$$\chi(x,y,z) = f(x,y)\,\omega(z)\,, \tag{2.11}$$

we see that Eq. (2.9) gives rise to two independent equations:

$$\left[\partial_x^2 + \partial_y^2 - 2\,i\,e\,A^2\,\partial_y - e^2\,(A_2)^2 + E_\perp \right] f(x,y) = 0 \tag{2.12}$$

and

$$\left[\partial_z^2 + i\,\xi\,\gamma^3\,\partial_z\,g(z) - \xi^2\,g^2(z) + E'^2 + i\,e\,B\,\gamma^1\,\gamma^2 \right]\omega(z) = 0\,, \tag{2.13}$$

with

$$E^2 = E'^2 + E_\perp\,. \tag{2.14}$$

Let us consider first Eq. (2.12). With the substitution

$$f(x,y) = e^{i\,p_y\,y}\,h(x)\,, \tag{2.15}$$

Eq. (2.12) becomes:

$$\left[-\frac{d^2}{dx^2} + p_y^2 - 2\,e\,B\,x\,p_y + e^2\,B^2\,x^2 \right] h(x) = E_\perp\,h(x)\,. \tag{2.16}$$

3

With the change of variable

$$\zeta = \sqrt{e\,B}\left(x - \frac{p_y}{eB}\right),\tag{2.17}$$

it is straightforward to show that the solutions of the Eq. (2.16) can be written in the form

$$h_n(\zeta) = A_n\,e^{-\frac{1}{2}\zeta^2}\,H_n(\zeta),\tag{2.18}$$

where $H_n(\zeta)$ is the $n-th$ Hermite polynomial and $A_n = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n!}}$. The energy is quantized according to:

$$E_{\perp,n} = e\,B\,(2\,n + 1)\,.\tag{2.19}$$

In order to solve Eq. (2.13), we expand $\omega(z)$ in terms of the eigenstates of γ^3:

$$\omega(z) = \phi_+^1\,u_+^1 + \phi_+^2\,u_+^2 + \phi_-^1\,u_-^1 + \phi_-^2\,u_-^2\tag{2.20}$$

where $u_\pm^s$ are given by

$$u_\pm^1 = \begin{pmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{pmatrix},\tag{2.21}$$

$$u_\pm^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \mp i \end{pmatrix},\tag{2.22}$$

which satisfy the following relations:

$$\begin{array}{rclcrcl}
\gamma^3 u_\pm^{1,2} &=& \pm i u_\pm^{1,2} & \quad & \gamma^1\,u_\pm^1 &=& \pm i\,u_\mp^2 \\
\gamma^0 u_\pm^{1,2} &=& +u_\mp^{1,2} & & \gamma^1\,u_\pm^2 &=& \mp i\,u_\mp^1 \\
\gamma^1\gamma^2 u_\pm^1 &=& -i u_\pm^1 & & \gamma^2\,u_\pm^1 &=& \mp u_\mp^2 \\
\gamma^1\gamma^2 u_\pm^2 &=& +i u_\pm^2 & & \gamma^2\,u_\pm^2 &=& \mp u_\mp^1\,.
\end{array}\tag{2.23}$$

With the aid of Eqs.(2.21)-(2.23), we recast Eq. (2.13) into:

$$\left(\frac{d^2}{dz^2} \mp \xi\frac{dg}{dz} - \xi^2 g^2(z) + E'^2 + e\,B\right)\phi_\pm^1 = 0\,,\tag{2.24}$$

$$\left(\frac{d^2}{dz^2} \mp \xi\frac{dg}{dz} - \xi^2 g^2(z) + E'^2 - e\,B\right)\phi_\pm^2 = 0\,.\tag{2.25}$$

For definiteness, let us consider Eq. (2.24). Following [19], we introduce the new variable

$$j = \frac{1}{1 + e^{2z}}.\tag{2.26}$$

It is easy to check that:

$$g(z) = 1 - 2j \, , \tag{2.27}$$

$$\frac{dg}{dz} = 4j \, (1 - j) \, , \tag{2.28}$$

and

$$\frac{d^2}{dz^2} = 4 \, j \, (1 - j) \left[(1 - 2j) \frac{d}{dj} + j \, (1 - j) \frac{d^2}{dj^2} \right] . \tag{2.29}$$

Eqs. (2.26)-(2.29) allow us to rewrite Eq. (2.24) as:

$$\left[\frac{d^2}{dj^2} + \frac{1 - 2j}{j(1 - j)} \frac{d}{dj} + \frac{E'^2 + eB \mp 4\xi \, j(1 - j) - 4 \, \xi^2 (1 - 2 \, j)^2}{4j^2(1 - j)^2} \right] \phi_{\pm}^1 = 0 \, . \tag{2.30}$$

By examining the behaviour of the differential equation near the singular points $j = 0$ and $j = 1$, we find:

$$\phi_{\pm}^1(z) \; = \; j^{\alpha^1} \, (1 \, - \, j)^{\beta^1} \, \chi_{\pm}^1(j) \, , \tag{2.31}$$

where $\chi_{\pm}^1(j)$ is regular near the singularities. A standard calculations gives:

$$\alpha^1 = \frac{i}{2} \sqrt{E'^2 + eB - \xi^2} \qquad , \quad \beta^1 = \alpha^1 \, . \tag{2.32}$$

Imposing that Eq. (2.31) is solution of Eq. (2.30), we see that $\chi_{\pm}^1(j)$ satisfies a hypergeometric equation [18] with parameters given by:

$$a_{\mp}^1 \; = \; 2\alpha^1 + \frac{1}{2} - \left| \xi \mp \frac{1}{2} \right| , \tag{2.33}$$

$$b_{\mp}^1 \; = \; 2\alpha^1 + \frac{1}{2} + \left| \xi \mp \frac{1}{2} \right| , \tag{2.34}$$

$$c^1 \; = \; 2\alpha^1 + 1 \, . \tag{2.35}$$

As well known, the general solution of the hypergeometric equation is given by the combination of the two independent solutions:

$$_2F_1(a_{\mp}, b_{\mp}, c, j) \, , \tag{2.36}$$

$$j^{1-c} \, _2F_1(a_{\mp} + 1 - c, b_{\mp} + 1 - c, 2 - c; j) \, . \tag{2.37}$$

Therefore Eq. (2.31) becomes:

$$\begin{aligned} \phi_{\pm}^1 \; &= \; A_{\pm}^1 \, j^{\alpha^1} (1 - j)^{\alpha^1} \, _2F_1(a_{\mp}^1, b_{\mp}^1, c^1, j) + \\ &\quad + B_{\pm}^1 \, j^{-\alpha^1} (1 - j)^{\alpha^1} \, _2F_1(a_{\mp}^1 + 1 - c^1, b_{\mp}^1 + 1 - c^1, 2 - c^1; j) \, , \end{aligned} \tag{2.38}$$

where $A_{\pm}^1$ and $B_{\pm}^1$ are normalization constants. For simplicity we define:

$$\phi_{\pm}^{(-\alpha^1)} \; = \; j^{\alpha^1} (1 - j)^{\alpha^1} \, _2F_1(a_{\mp}^1, b_{\mp}^1, c^1; j) \, , \tag{2.39}$$

$$\phi_{\pm}^{(+\alpha^1)} \; = \; j^{-\alpha^1} (1 - j)^{\alpha^1} \, _2F_1(a_{\mp}^1 + 1 - c^1, b_{\mp}^1 + 1 - c^1, 2 - c^1; j) \, . \tag{2.40}$$

Then Eq. (2.38) can be written as:

$$\phi_\pm^1 = A_\pm^1 \, \phi_\pm^{(-\alpha^1)} + B_\pm^1 \, \phi_\pm^{(+\alpha^1)}. \tag{2.41}$$

Therefore, assuming $\omega(z) = \phi_+^1 \, u_+^1$, Eq. (2.11) becomes:

$$\chi(x, y, z) = A_n \, e^{-\frac{1}{2}\zeta^2 + i \, p_y \, y} \, H_n(\zeta) \, [A_+^1 \, \phi_+^{(-\alpha^1)} + B_+^1 \, \alpha_+^{(+\alpha^1)}] \, u_+^1. \tag{2.42}$$

Finally, the general solution the can be obtained from Eq. (2.5):

$$
\begin{aligned}
\Psi_r^1(x, y, z, t) &= [(i\gamma^0 \, \partial_t + \xi g(z) + i\gamma^3 \partial_z) \, \phi_+^1 \, u_+^1 \, e^{-iE_r t}] \, f(x, y) + \\
&\quad [(i\gamma^1 \partial_x + i\gamma^2 \partial_y - e\gamma^2 A_2) f(x, y)] \, \phi_+^1 \, u_+^1 \, e^{-i \, E_r \, t} \\
&\equiv \psi_r(z, t) \, f(x, y) + \eta(x, y) \, \phi_+^1 \, u_+^1 \, e^{-i \, E_r \, t}
\end{aligned}
\tag{2.43}
$$

where

$$\psi_r(z, t) \equiv [i\gamma^0 \, \partial_t + \xi g(z) + i\gamma^3 \partial_z] \, \phi_+^1 \, u_+^1 \, e^{-iE_r t} \tag{2.44}$$

and

$$\eta(x, y) \equiv (i\gamma^1 \partial_x + i\gamma^2 \partial_y - e\gamma^2 A_2) \, A_n \, H_n(\zeta) \, e^{-\frac{1}{2}\zeta^2 + i \, p_y \, y} \, . \tag{2.45}$$

In order to esplicitate $\psi_r(z, t)$, let us assume that

$$\varphi_\pm^1 = \left[\mp \frac{d}{dz} + \xi \, g(z) \right] \phi_\pm^1 \, , \tag{2.46}$$

so that Eq. (2.44) becomes:

$$\psi_r(z, t) = (E_r \, \phi_+^1 \, u_-^1 + \varphi_+^1 \, u_+^1) \, e^{-iE_r t}. \tag{2.47}$$

A calculation similar to the one performed in the Appendix of Ref. [19] gives:

$$\varphi_\pm^{(-\alpha^1)} = (\xi \pm 2\,\alpha^1) \, \phi_\mp^{(-\alpha^1)} \tag{2.48}$$

and

$$\varphi_\pm^{(+\alpha^1)} = (\xi \mp 2\,\alpha^1) \, \phi_\mp^{(+\alpha^1)} \, . \tag{2.49}$$

We have then:

$$
\begin{aligned}
\psi_r(z, t) &= A^1 \, [E_r \, \phi_+^{1 \, (-\alpha^1)} u_-^1 + (\xi + 2\,\alpha^1)\phi_-^{1 \, (-\alpha^1)} \, u_+^1] + \\
&\quad B^1 \, [E_r \, \phi_+^{1 \, (+\alpha^1)} u_-^1 + (\xi - 2\,\alpha^1)\phi_-^{1 \, (+\alpha^1)} \, u_+^1] \, e^{-i \, E_r \, t}.
\end{aligned}
\tag{2.50}
$$

We are now interested in the calculation of the reflection and transmission coefficients for fermions incident on the wall from $z \to -\infty$ to $z \to +\infty$. To this end, it is necessary to consider the asymptotic forms of $\phi_\pm^{(-\alpha^1)}$ and $\phi_\pm^{(+\alpha^1)}$:

$$\lim_{z \to +\infty} j^{\pm\alpha^1} (1 - j)^{\alpha^1} = \exp(\mp 2\,\alpha^1 \, z) \, , \tag{2.51}$$

6

$$\lim_{z \to -\infty} j^{\alpha^1} (1-j)^{\pm \alpha^1} = \exp(\pm 2\,\alpha^1 z)\,. \tag{2.52}$$

Equation (2.50) has two terms with different asymptotic properties for $z \to +\infty$ and $z \to -\infty$. It is simple to see that the boundary conditions for $\psi_r(z)$ imply $A^1 = 0$. As a consequence we have:

$$\psi_r(z,t) = B^1 \left[E_r\, \phi_+^{1\,(+\alpha^1)}\, u_-^1 + (\xi - 2\,\alpha^1)\, \phi_-^{1\,(+\alpha^1)}\, u_+^1 \right] e^{-i\,E_r\,t}\,. \tag{2.53}$$

Concerning $\eta(x,y)$, since

$$\frac{d}{dx} = \sqrt{eB}\,\frac{d}{d\zeta}\,, \qquad\qquad \frac{dH_n(\zeta)}{d\zeta} = 2\,n\,H_{n-1}(\zeta)\,, \tag{2.54}$$

we get from Eq. (2.45)

$$\eta(x,y) = A_n \left\{ i\,\gamma^1\,\sqrt{eB}\,[2\,n\,H_n(\zeta) - \zeta\,H_n(\zeta)] - (p_y - eBx)\,\gamma^2\,H_n(\zeta) \right\} e^{i p_y y - \frac{1}{2}\zeta^2}\,. \tag{2.55}$$

Observing that $\omega(z) = \phi_+^1\, u_+^1$ and taking into account Eq. (2.23), we obtain

$$\eta(x,y)\,\phi_+^1\, u_+^1\, e^{-i\,E_r\,t} = A_n\,[(\zeta\,\sqrt{eB} + p_y - eBx)\,H_n(\zeta) - 2n\,\sqrt{eB}\,H_{n-1}(\zeta)]\,\phi_+^1\, u_-^2\, e^{i p_y y - \frac{1}{2}\zeta^2 - iE_r t}\,. \tag{2.56}$$

We can note now that the coefficient of H_n in the last equation vanishes. So that we get:

$$\begin{aligned}
\Psi_r(x,y,z,t) = {}& A_n\,B^1\,\Big\{ H_n(\zeta)\,[E_r\,\phi_+^{(+\alpha^1)}\, u_-^1 + (\xi - 2\,\alpha^1)\,\phi_-^{(+\alpha^1)}\, u_+^1] + \\
& -2n\sqrt{eB}\,H_{n-1}\,\phi_+^{(+\alpha^1)}\, u_-^2 \Big\}\, e^{i p_y y - \frac{1}{2}\zeta^2 - iE_r t}\,.
\end{aligned} \tag{2.57}$$

Writing

$$A_r(x,y,t) = A_n\,B^1\, e^{i p_y y - \frac{1}{2}\zeta^2 - iE_r t}\,, \tag{2.58}$$

the general solution becomes:

$$\begin{aligned}
\Psi_{r,n}^1(x,y,z,t) = A_r(x,y,t) \quad \big\{ &\; [E_r\,H_n(\zeta)\,u_-^1 - 2n\sqrt{eB}\,H_{n-1}(\zeta)\,u_-^2]\,\phi_+^{(+\alpha^1)} \\
&+ H_n(\zeta)\,(\xi - 2\alpha^1)\,u_+^1\,\phi_-^{(+\alpha^1)} \big\}\,.
\end{aligned} \tag{2.59}$$

To obtain the asymptotic behaviour of the wave function, we use Eq. (2.39) to get:

$$\begin{aligned}
\Psi_r(x,y,z,t) = {}& A_r(x,y,t)\{[E_r\,H_n(\zeta)\,u_-^1 - 2n\sqrt{eB}\,H_{n-1}(\zeta)\,u_-^2]\, {}_2F_1(-\xi + 1, \xi, 1 - 2\,\alpha^1; j) + \\
& + H_n(\zeta)\,(\xi - 2\alpha^1)\,u_+^1\, {}_2F_1(-\xi, \xi + 1, 1 - 2\,\alpha^1; j)\}\,j^{-\alpha^1}\,(1-j)^{\alpha^1}\,.
\end{aligned} \tag{2.60}$$

In order to find the expansion for $z \to -\infty$ (i.e. $j \to 1$), we need to consider the analytical extension of the hypergeometric functions by means of the Kummer's formula [18]:

$$\begin{aligned}
F(a,b,c;z) = {}& \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}\,F(a,b,a+b-c+1,z) + (1-z)^{c-a-b}\,\cdot \\
& \cdot \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}\,F(c-a,c-b,c-a-b+1,1-z)\,.
\end{aligned} \tag{2.61}$$

By means of the well known relation:

$$\Gamma(z+1) = z\,\Gamma(z) , \qquad (2.62)$$

and after some manipulations we obtain the transmitted, incident and reflected wave functions:

$$\left(\ \Psi^1_{r,n}(x,y,z,t)\right)^{tran} = A_r(x,y,t)[\,H_n(\zeta)\,E_r\,u^1_- - 2n\sqrt{eB}H_{n-1}(\zeta)\,u^2_- + H_n(\zeta)\,(\xi - 2\,\alpha^1)\,u^1_+]\,e^{2\alpha^1\,z} \qquad (2.63)$$

$$\left(\ \Psi^1_{r,n}(x,y,z,t)\right)^{inc} = A_r(x,y,t)\frac{\Gamma(1-2\,\alpha^1)\,\Gamma(-2\,\alpha^1)}{\Gamma(-2\,\alpha^1+\xi)\,\Gamma(-2\,\alpha^1-\xi)}\,e^{2\alpha^1\,z}\left[\frac{E_r\,H_n(\zeta)}{-2\,\alpha^1-\xi}\,u^1_- + \right.$$
$$\left. + H_n(\zeta)\,u^1_+ - \frac{2\,n\sqrt{eB}\,H_{n-1}(\zeta)}{-2\,\alpha^1-\xi}\,u^2_-\right] \qquad (2.64)$$

$$\left(\ \Psi^1_{r,n}(x,y,z,t)\right)^{refl} = A_r(x,y,t)\frac{\Gamma(1-2\,\alpha^1)\,\Gamma(2\,\alpha^1)}{\Gamma(\xi)\,\Gamma(-\xi)}\,e^{-2\alpha^1\,z}\left[\frac{E_r\,H_n(\zeta)}{-\xi}\,u^1_- + \right.$$
$$\left. + \frac{H_n(\zeta)\,(\xi - 2\,\alpha^1)}{\xi}\,u^1_+ - \frac{2\,n\sqrt{eB}\,H_{n-1}(\zeta)}{-\xi}\,u^2_-\right] . \qquad (2.65)$$

Making use of the following relations:

$$(u^1_-)^\dagger\gamma^0\gamma^3 u^1_- = 0 \qquad (u^1_+)^\dagger\gamma^0\gamma^3 u^1_- = -2\,i \qquad (u^2_-)^\dagger\gamma^0\gamma^3 u^1_- = 0$$

$$(u^1_-)^\dagger\gamma^0\gamma^3 u^1_+ = 2\,i \qquad (u^1_+)^\dagger\gamma^0\gamma^3 u^1_+ = 0 \qquad (u^2_-)^\dagger\gamma^0\gamma^3 u^1_+ = 0 \qquad (2.66)$$

$$(u^1_-)^\dagger\gamma^0\gamma^3 u^2_- = 0 \qquad (u^1_+)^\dagger\gamma^0\gamma^3 u^2_- = 0 \qquad (u^2_-)^\dagger\gamma^0\gamma^3 u^2_- = 0 ,$$

it is straightforward to obtain the currents:

$$(j^3_V)^{inc} = \frac{8}{\pi^2}i\,\alpha^1\,E_r|A_r(x,y,t)|^2\,H^2_n(\zeta)\,|\Gamma(1+2\alpha^1)\Gamma(2\alpha^1)|^2\,\sin[\pi(2\alpha^1+\xi)]\sin[\pi(2\alpha^1-\xi)], \qquad (2.67)$$

$$(j^3_V)^{tran} = -8\,i\,\alpha^1\,|A_r(x,y,t)|^2\,H^2_n(\zeta)\,E_r , \qquad (2.68)$$

$$(j^3_V)^{refl} = \frac{8}{\pi^2}i\,\alpha^1\,E_r|A_r(x,y,t)|^2\,H^2_n(\zeta)\,|\Gamma(1+2\alpha^1)\Gamma(2\alpha^1)|^2\,\sin^2(\pi\,\xi) . \qquad (2.69)$$

By means of the identity [18]

$$\Gamma(z)\Gamma(-z) = \frac{-\pi}{z\sin(\pi z)} , \qquad (2.70)$$

we finally obtain the following reflection and transmission coefficients:

$$R^1 = \frac{-(j_V^3)^{refl}}{(j_V^3)^{inc}} = \frac{\sin^2(\pi\,\xi)}{\sin[\pi(\xi - 2\,\alpha^1)]\,\sin[\pi(\xi + 2\,\alpha^1)]}\,, \tag{2.71}$$

$$T^1 = \frac{(j_V^3)^{tran}}{(j_V^3)^{inc}} = \frac{-\sin^2(2\pi\,\alpha^1)}{\sin[\pi(\xi - 2\,\alpha^1)]\,\sin[\pi(\xi + 2\,\alpha^1)]}\,. \tag{2.72}$$

It is worthwhile to stress that the dependence of the reflection and transmittion coefficients on the magnetic field is encoded into α^1. Indeed, from Eqs. (2.14), (2.19), and (2.32) we have:

$$\alpha^1 = \frac{i}{2}\sqrt{E^2 - 2neB - \xi^2} \qquad n = 0, 1, 2... \quad . \tag{2.73}$$

In the same way we can solve the case of fermions with spin projection on the third spatial axis antiparallel to the magnetic field. In this case the relevant equation turns out to be Eqs. (2.24) and (2.25). Following the same steps of the previous derivation, we find:

$$R^2 = \frac{\sin^2(\pi\,\xi)}{\sin[\pi(\xi - 2\,\alpha^2)]\,\sin[\pi(\xi + 2\,\alpha^2)]}\,, \tag{2.74}$$

$$T^2 = \frac{-\sin^2(2\pi\,\alpha^2)}{\sin[\pi(\xi - 2\,\alpha^2)]\,\sin[\pi(\xi + 2\,\alpha^2)]}\,, \tag{2.75}$$

where now:

$$\alpha^2 = \frac{i}{2}\sqrt{E^2 - 2eB(n+1) - \xi^2} \qquad n = 0, 1, 2... \quad . \tag{2.76}$$

It is interesting to note that the dependence due to the motion in the direction transverse to the magnetic field plane factorizes in the expression of the currents, in such a way that the reflection and transmission coefficients do not show any explicit (x, y) dependence. Of course, the reflection and transmission coefficients depend on the Landau level index n.

In Fig. 1 we display the reflection coefficient for parallel spin R^1 as a function of the scaled energy E/ξ, with fixed magnetic field at various values of n. We see that there is total reflection for fermions with parallel spin at energies $E^2 - \xi^2 = 2neB$. In the case of antiparallel spin we find that the total reflection occurs at energies $E^2 - \xi^2 = 2(n+1)eB$. As we shall discuss in the next Section, this peculiar anomalous scattering can be understood as due to the presence of solutions localized on the kink domain wall.

Figure 2 shows both the reflection coefficients for parallel and antiparallel spin as a function of the magnetic field at fixed energy and two different values of Landau level index n. Note that R^1 for $n = 0$ is independent of the magnetic field. We see that the magnetic field is able to produce an asymmetry in the spin distribution, but there is no particle-antiparticle asymmetry, which would be relevant in the electroweak baryogenesis. Moreover, we see that the difference between these coefficients grows with an increasing field strength.

Finally, in Fig. 3 we compare the transmission coefficients for parallel and antiparallel spin as a function of the magnetic field at fixed energy and two different values of Landau level index n. Note that, as expected, we have $R + T = 1$.

III.   SOLUTIONS LOCALIZED ON THE WALL

In this Section we discuss fermion states corresponding to solutions of the Dirac equation (2.4) localized on the domain wall. Following the approach of the previous Section, we find again Eqs. (2.24) and (2.25), that for completeness we rewrite here:

$$\left(\frac{d^2}{dz^2} \mp \xi \frac{dg(z)}{dz} - \xi^2 g^2(z) + E'^2 + eB\right)\phi_\pm^1 = 0 \,, \tag{3.1}$$

$$\left(\frac{d^2}{dz^2} \mp \xi \frac{dg(z)}{dz} - \xi^2 g^2(z) + E'^2 - eB\right)\phi_\pm^2 = 0 \,, \tag{3.2}$$

where, according to Eqs. (2.14) and (2.19),

$$E'^2 = E^2 - eB\,(2\,n+1) \,. \tag{3.3}$$

It is known since long time that in the absence of magnetic field these differential equations admit zero energy solutions localized on the wall [20, 21]. In our case, we see that the condition to localize fermions on the wall is given by:

$$E'^2 + eB = 0 \quad \text{for} \quad \phi_\pm^1 \,, \tag{3.4}$$

$$E'^2 - eB = 0 \quad \text{for} \quad \phi_\pm^2 \,, \tag{3.5}$$

namely,

$$E^2 = 2neB \quad \text{for} \quad \phi_\pm^1 \,, \tag{3.6}$$

$$E^2 = 2(n+1)eB \quad \text{for} \quad \phi_\pm^2 \,. \tag{3.7}$$

In Section 2 we found that there is total reflection for fermions with parallel and antiparallel spin at energies $E^2 - \xi^2 = 2neB$ and $E^2 - \xi^2 = 2(n+1)eB$, respectively. The difference is due to the fermion mass ξ which, indeed, vanishes on the wall where the system is in a symmetric phase. From a physical point of view, we see that fermions with asymptotically non-zero momentum $|\vec{p}|$ equal to $\sqrt{2neB}$ for parallel spin and $\sqrt{2(n+1)eB}$ for antiparallel spin can be trapped on the domain wall.

Let us now discuss the localized solutions. Inserting Eqs. (3.4) and (3.5) into Eqs. (3.1) and (3.2), respectively, we get:

$$\left(\frac{d^2}{dz^2} \mp \xi \frac{dg(z)}{dz} - \xi^2 g^2(z)\right)\phi_\pm^{1,2} = 0 \,. \tag{3.8}$$

It is easy to find the solutions of Eq. (3.8):

$$\phi_\pm^{1,2}(z) = N \exp\left[\mp\xi \int_0^z dz'\, g(z')\right] \,, \tag{3.9}$$

with N normalization constant. Clearly, the $\phi_+^{1,2}$ solutions must be neglected because they are divergent for $|z| \to +\infty$. We are left with the $\phi_-^{1,2}$ solutions, explicitly given by:

$$\phi_-^{1,2}(z) = N \, (\cosh z)^{-\xi}. \tag{3.10}$$

In order to evaluate the localized states, we insert Eq. (3.10) into Eq. (2.5) and take care of Eqs. (2.7), (2.11) and (2.20) to get:

$$\psi_{loc}^{1,2}(x,y,z,t) = (i\gamma^\mu \partial_\mu + \xi \, g(z) - e\gamma^\mu A_\mu) \, f(x,y) \, \phi_-^{1,2}(z) \, u_-^{1,2} \, e^{-i\,Et} \, . \tag{3.11}$$

After some manipulations we obtain:

$$\psi_{loc}^1(x,y,z,t) = N \, A_n \, e^{-\frac{\zeta^2}{2}+i\,p_y\,y-i\,E\,t} \, (\cosh z)^{-\xi} \, [H_n \, E \, u_+^1 + 2\,n\,\sqrt{eB}\,H_{n-1}\,u_+^2] \tag{3.12}$$

and

$$\psi_{loc}^2(x,y,z,t) = N \, A_n \, e^{-\frac{\zeta^2}{2}+i\,p_y\,y-i\,E\,t} \, (\cosh z)^{-\xi} \, [H_n \, E \, u_+^2 + \sqrt{eB}\,H_{n+1}\,u_+^1] \, , \tag{3.13}$$

where A_n has been defined in Sec 2. Of course, $E^2 = 2\,n\,e\,B$ in Eq. (3.12), while $E^2 = 2\,(n+1)\,e\,B$ in Eq. (3.13).
The normalization condition

$$\int d^3x \, \psi_{loc}^\dagger \, \psi_{loc} = \delta(p_y - p_y') \, \delta_{nn'} \tag{3.14}$$

gives

$$N^2 = \frac{\sqrt{eB}}{8\,\pi\,B(\xi,1/2)\,E^2} \, , \tag{3.15}$$

where $B(x,y)$ is the Bernoulli beta function [18].
It is interesting to note that our localized solutions can be rewritten in the equivalent form:

$$\psi_{loc}(x,y,z,t) \;=\; N \begin{pmatrix} v(x,y) \\ i\,\sigma^3\,v(x,y) \end{pmatrix} (\cosh z)^{-\xi}\,e^{-i\,E\,t} \, , \tag{3.16}$$

where, for instance, in the case of the localized wave function ψ_{loc}^1 we find

$$v(x,y) = A_n \, e^{i\,p_y\,y-\frac{\zeta^2}{2}} \begin{pmatrix} H_n(\zeta) \\ \frac{2\,n\,\sqrt{eB}}{E_r}\,H_{n-1}(\zeta) \end{pmatrix} \, . \tag{3.17}$$

A similar result holds for ψ_{loc}^2.

IV. CONCLUSIONS

Much work has been recently devoted to study effects due to scattering of particles from domain walls between the phases of broken and unbroken symmetry at the electroweak phase transition [19, 22–24]. The main effort of this paper consists in the investigation of the effects of a constant magnetic field on the scattering of fermions on planar walls. In particular we focused our attention on kink domain walls, which are of interest in a

continuum cosmological phase transition. We solved the Dirac equation for scattering of fermions off domain walls, and computed the transmission and reflection coefficients. As expected, we find that the constant magnetic field induces a spin asymmetry in fermion reflection and transmission.

The results obtained, as a matter of fact, do not allow to produce directly asymmetries in some local charges, which is a prerequisite for electroweak baryogenesis [9–11]. However, we would like to stress that, in the physical condition of early Universe, fermions moving through the domain walls will interact not only with the wall but also with the particles in the surrounding plasma. So that, even though we expect in general that an asymmetry between fermion and antifermion distributions in the primordial plasma could be induced by magnetic fields, such analysis requires the study of quantum Boltzmann transport equation [25, 26].

An interesting aspect of kink domain walls, not shared by bubble domain walls, is the presence of fermion states localized on the wall, effect due to the magnetic field. One could speculate that for these trapped fermions the sphaleron mechanism could efficiently converts any local charge asymmetry into a baryon number asymmetry. We deserve such an analysis to a future work.

Aside from these considerations, kink domains walls with the associated magnetic field could display interesting cosmological properties. Let us then conclude this paper by briefly discussing the role of the domain walls in the early Universe. As we have anticipated in the Introduction, the possible role of kink domain walls in the cosmological dynamics has been so far neglected, due to the fact that the existence of such walls was ruled out by the Zel'dovich, Kobazarev and Okun [12] argument. However, as we argued before, ferromagnetic domain walls with vanishing effective surface tension [13, 14] are an open possibility.

It is interesting to stress that the same mechanism which leads to a vanishing effective surface tension of the domain walls gives rise to a vanishing effective energy-momentum tensor. However, in Section 3 we have seen that kink domains in a constant magnetic field display positive energy fermion states localized on the wall. If one takes into account the contribution of these trapped fermions to the energy-momentum tensor, then one finds that the kink domain wall acquires a non vanishing traceless energy-momentum tensor. As a consequence, the gravitational dynamics of such domain walls is fully relativistic. We shall report progress on this subject in a forthcoming work [27].

APPENDIX

A   Bubble Walls

In this Appendix we discuss fermions scattering off bubble walls in a constant magnetic field. This case is relevant for a first order phase transition where the conversion from one phase to another takes place through nucleation. The region separating the two phases is considered as the wall. Under the assumption that the wall is thin and that the phase transition takes place when the energy densities of both the phases are degenerate, it is possible to approximate the wall profile in the form

$$\varphi(z) = \frac{v}{2}[1 + \tanh z] \,, \tag{A.1}$$

where we set the bubble wall thickness $\Delta = 1$. We see that $z < 0$ represents the region outside the bubble, i.e. the region in the symmetric phase where particles are massless. Conversely, for $z > 0$, the system is inside the bubble, i.e. in the broken phase and the particles have acquired a finite mass. When scattering is not affected by diffusion, the problem of fermion reflection and transmission through the wall can be casted in terms of solving the Dirac equation with a position dependent fermion mass, proportional to the Higgs field [19].

Recently, it has been shown that in the presence of primordial hypermagnetic fields it is possible to generate an axial asymmetry during the reflection and transmission of fermions off bubble walls [28]. This result has been obtained with the same approximation used in our previous work [16], i.e. by considering solutions describing the motion od fermions perpendicular to the wall. In this Appendix we include the effects due to the motion of fermions parallel to the wall in the presence of a constant magnetic field. However, our results can be easily extended to the case of hypermagnetic fields discussed in Ref. [28]. Starting from the Dirac equation (2.2) and assuming Eq. (2.6), we obtain:

$$[-\partial^2 + i\,\xi\gamma^3\partial_z\,g(z) - 2i\,e\,A^\mu\partial_\mu - i\,e\partial^\mu\,A_\mu - \frac{e}{2}\,\sigma^{\mu\nu}\,F_{\mu\nu} +$$
$$-\xi^2\,g^2(z) + e^2\,A_\mu A^\mu]\,\Phi(x,y,z,t) = 0 \,, \tag{A.2}$$

where now:

$$g(z) = \frac{v}{2}[1 + \tanh z] \,, \tag{A.3}$$

and 2ξ is the mass that the fermion acquires in the broken phase.

Using Eq. (2.10) and factorizing $\Phi(x,y,z,t)$ according to Eqs. (2.7) and (2.11), we get:

$$[\,\partial_x^2 + \partial_y^2 - 2\,i\,e\,A^2\,\partial_y - e^2\,(A_2)^2 + E_\perp\,]\,f(x,y) = 0 \tag{A.4}$$

and

$$[\,\partial_z^2 + i\,\xi\,\gamma^3\,\partial_z\,g(z) - \xi^2\,g^2(z) + E'^2 + i\,e\,B\,\gamma^1\,\gamma^2\,]\,\omega(z) = 0 \tag{A.5}$$

with

$$E^2 = E'^2 + E_\perp \,. \tag{A.6}$$

Eq. (A.4) agrees with Eq. (2.12). So that $f(x,y)$ is given by Eqs. (2.14) and (2.18), and $E_\perp$ is quantized according to Eq. (2.19).

In order to solve Eq. (A.4), we expand $\omega(z)$ in terms of the spinors $u_\pm^s$ eigenstates of γ^3, Eqs. (2.21) and (2.22). We get:

$$\left(\frac{d^2}{dz^2} \mp \xi \frac{dg}{dz} - \xi^2 g^2(z) + E'^2 + eB \right) \phi_\pm^1 = 0 \tag{A.7}$$

$$\left(\frac{d^2}{dz^2} \mp \xi \frac{dg}{dz} - \xi^2 g^2(z) + E'^2 - eB \right) \phi_\pm^2 = 0 . \tag{A.8}$$

For definiteness, let us consider Eq. (A.7). Introducing the new variable

$$j = \frac{1}{1 + e^{2z}} , \tag{A.9}$$

we obtain

$$\left[\frac{d^2}{dj^2} + \frac{1 - 2j}{j(1 - j)} \frac{d}{dj} + \frac{E'^2 + eB \mp 4\xi j(1 - j) - 4\xi^2(1 - j)^2}{4j^2(1 - j)^2} \right] \phi_\pm^1 = 0 . \tag{A.10}$$

The behaviour of the differential equation near the singular points allows us to write

$$\phi_\pm^1(z) = j^{\alpha^1} (1 - j)^{\beta^1} \chi_\pm^1(j) , \tag{A.11}$$

where

$$\alpha^1 = \frac{i}{2} \sqrt{E^2 - 2neB - 4\xi^2} , \tag{A.12}$$

$$\beta^1 = \frac{i}{2} \sqrt{E^2 - 2neB} . \tag{A.13}$$

It is easy to see that $\chi_\pm^1(j)$ satisfy the hypergeometric equation [18] with parameters:

$$\begin{aligned}
a_\mp^1 &= \alpha^1 + \beta^1 + \frac{1}{2} - \left| \xi \mp \frac{1}{2} \right| , \\
b_\mp^1 &= \alpha^1 + \beta^1 + \frac{1}{2} + \left| \xi \mp \frac{1}{2} \right| , \\
c^1 &= 2\alpha^1 + 1 .
\end{aligned} \tag{A.14}$$

Concerning the solution of Eq. (A.8), it is straightforward to see that $\phi_\pm^2$ can be obtained from $\phi_\pm^1$ provided α^1 and β^1 are replaced by

$$\alpha^2 = \frac{i}{2} \sqrt{E^2 - 2(n+1)eB - 4\xi^2} , \tag{A.15}$$

$$\beta^2 = \frac{i}{2} \sqrt{E^2 - 2(n+1)eB} . \tag{A.16}$$

14

In order to write the most general solutions, it is useful to introduce the following notations:

$$\epsilon_s = \begin{cases} +1 \text{ if } s=1 \\ -1 \text{ if } s=2 \end{cases}, \quad h_s = \begin{cases} 2n\frac{H_{n-1}}{H_n} \text{ if } s=1 \\ \frac{H_{n+1}}{H_n} \text{ if } s=2 \end{cases}, \quad \bar{s} = \begin{cases} +1 \text{ if } s=2 \\ -1 \text{ if } s=1 . \end{cases} \tag{A.17}$$

After some calculations similar to those performed in Section 2, we obtain the transmitted, incident and reflected wave functions:

$$(\Psi_{n,r,s}^{\pm})^{tran} = A_r\, H_n[E_r\, u_{\mp}^s + 2(\xi \mp \alpha^s)u_{\pm}^s \mp \sqrt{eB}\, h_s\, u_{\mp}^{\bar{s}}] \exp(2\,\alpha^s\, z)\,, \tag{A.18}$$

$$(\Psi_{n,r,s}^{\pm})^{inc} = A_r\, H_n\, \frac{\Gamma(1-2\,\alpha^s)\Gamma(-2\beta^s)}{\Gamma(-\alpha^s-\beta^s+\xi)\,\Gamma(-\alpha^s-\beta^s-\xi)} \frac{1}{-\alpha^s-\beta^s\mp\xi}$$
$$\left[E_r u_{\mp}^s + \frac{2(\xi\mp\alpha^s)(-\alpha^s-\beta^s\mp\xi)}{-\alpha^s-\beta^s\pm\xi}\, u_{\pm}^s \mp \sqrt{eB}\, h_s\, u_{\mp}^{\bar{s}}\right] e^{2\beta^s z}\,, \tag{A.19}$$

$$(\Psi_{n,r,s}^{\pm})^{refl} = A_r\, H_n\, \frac{\Gamma(1-2\,\alpha^s)\Gamma(2\beta^s)}{\Gamma(-\alpha^s+\beta^s-\xi)\,\Gamma(-\alpha^s+\beta^s+\xi)} \frac{1}{-\alpha^s+\beta^s\mp\xi}$$
$$\left[E_r u_{\mp}^s + \frac{2(\xi\mp\alpha^s)(-\alpha^s+\beta^s\mp\xi)}{-\alpha^s+\beta^s\pm\xi}\, u_{\pm}^s \mp \sqrt{eB}\, h_s\, u_{\mp}^{\bar{s}}\right] e^{-2\beta^s z}\,. \tag{A.20}$$

Likewise, the transmitted, incident and reflected currents are:

$$(j_{V,s}^3)^{tran} = 8\,\epsilon_s\, E_r\, H_n^2\, |A_r|^2\, |\alpha^s| \tag{A.21}$$

$$(j_{V,s}^3)^{inc} = \frac{-8}{\pi^2}\epsilon_s E_r H_n^2 |A_r|^2\, |\alpha^s| |\Gamma(1+2\alpha^s)\Gamma(2\beta^s)|^2\, \sin[\pi(\xi+\alpha^s+\beta^s)]\, \sin[\pi(\xi-\alpha^s-\beta^s)] \tag{A.22}$$

$$(j_{V,s}^3)^{refl} = \frac{8}{\pi^2}\epsilon_s E_r H_n^2 |A_r|^2\, |\alpha^s| |\Gamma(1+2\alpha^s)\Gamma(2\beta^s)|^2\, \sin[\pi(\xi+\alpha^s-\beta^s)]\, \sin[\pi(\xi-\alpha^s+\beta^s)]\,. \tag{A.23}$$

Therefore the reflection and transmission coefficients are given by:

$$R^s = \frac{\sin[\pi(\alpha^s-\beta^s+\xi)]\, \sin[\pi(-\alpha^s+\beta^s+\xi)]}{\sin[\pi(-\alpha^s-\beta^s+\xi)]\, \sin[\pi(+\alpha^s+\beta^s+\xi)]}\,, \tag{A.24}$$

$$T^s = \frac{-\sin(2\pi\,\alpha^s)\, \sin(2\pi\beta^s)}{\sin[\pi(-\alpha^s-\beta^s+\xi)]\, \sin[\pi(+\alpha^s+\beta^s+\xi)]}\,. \tag{A.25}$$

15

References

[1] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 1994).

[2] T. W. B. Kibble, J. Phys. **A9**, 1387 (1976).

[3] T. W. B. Kibble, Phys. Rept. **67**, 183 (1980).

[4] W. H. Zurek, Nature **317**, 505 (1985).

[5] W. H. Zurek, Phys. Rept. **276**, 177 (1996).

[6] ALEPH, R. Barate *et al.*, Phys. Lett. **B499**, 53 (2001).

[7] Z. Fodor, Nucl. Phys. Proc. Suppl. **83**, 121 (2000).

[8] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, Phys. Rev. Lett. **77**, 2887 (1996).

[9] M. Trodden, Rev. Mod. Phys. **71**, 1463 (1999).

[10] A. Riotto, (1998), hep-ph/9807454.

[11] A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49**, 35 (1999).

[12] Y. B. Zeldovich, I. Y. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3 (1974).

[13] P. Cea and L. Tedesco, Phys. Lett. **B450**, 61 (1999).

[14] P. Cea and L. Tedesco, J. Phys. **G26**, 411 (2000).

[15] T. Vachaspati, Phys. Lett. **B265**, 258 (1991).

[16] P. Cea, G. L. Fogli, and L. Tedesco, Mod. Phys. Lett. **A15**, 1755 (2000).

[17] J. D .Bjorken and S. Drell, *Relativistic Quantum Fields* (McGraw Hill, New York, 1962).

[18] I. S. Gradshteyn and I. M. Ryzshik, *Table of Integrals, Series and Products* (Academic Press, New York, 1963).

[19] A. Ayala, J. Jalilian-Marian, L. McLerran, and A. P. Vischer, Phys. Rev. **D49**, 5559 (1994).

[20] R. Jackiw and C. Rebbi, Phys. Rev. **D13**, 3398 (1976).

[21] A. J. Niemi and G. W. Semenoff, Phys. Rept. **135**, 99 (1986).

[22] K. Funakubo, A. Kakuto, S. Otsuki, K. Takenaga, and F. Toyoda, Phys. Rev. **D50**, 1105 (1994).

[23] G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. **D50**, 774 (1994).

[24] G. R. Farrar and John W. McIntosh, Phys. Rev. **D51**, 5889 (1995).

[25] L. P. Kadanoff and A. Baym, *Quantum Statistical Mechanics* (W. A. Benjamin, 1962).

[26] E. M .Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon Press, 1979).

[27] L. Campanelli, P. Cea, G. L. Fogli, and L. Tedesco, *Dynamics of Ferromagnetic Walls: Gravitational Properties* , in preparation.

[28] A. Ayala, J. Besprosvany, G. Pallares, and G. Piccinelli, (2001), hep-ph/0107072.

List of Figures

Figure 1: Reflection coefficient for parallel spin versus the scaled energy for four different values of the Landau level index n with $eB = 0.1$.
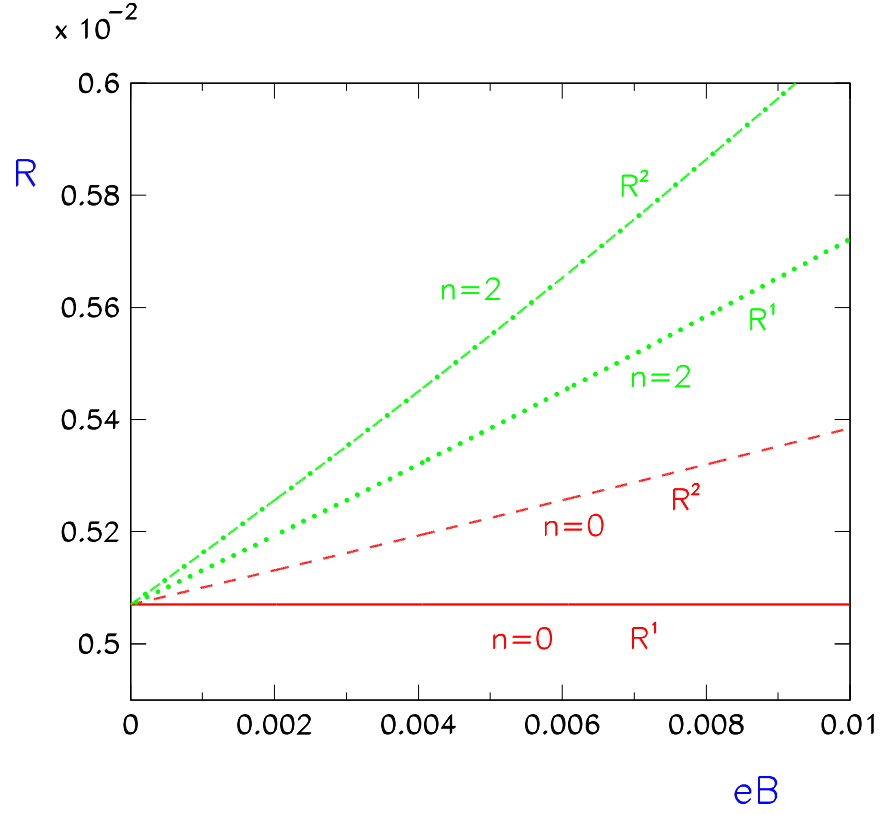
Figure 2: Reflection coefficients for parallel and antiparallel spin at fixed energy as a function of eB for two different values of n.
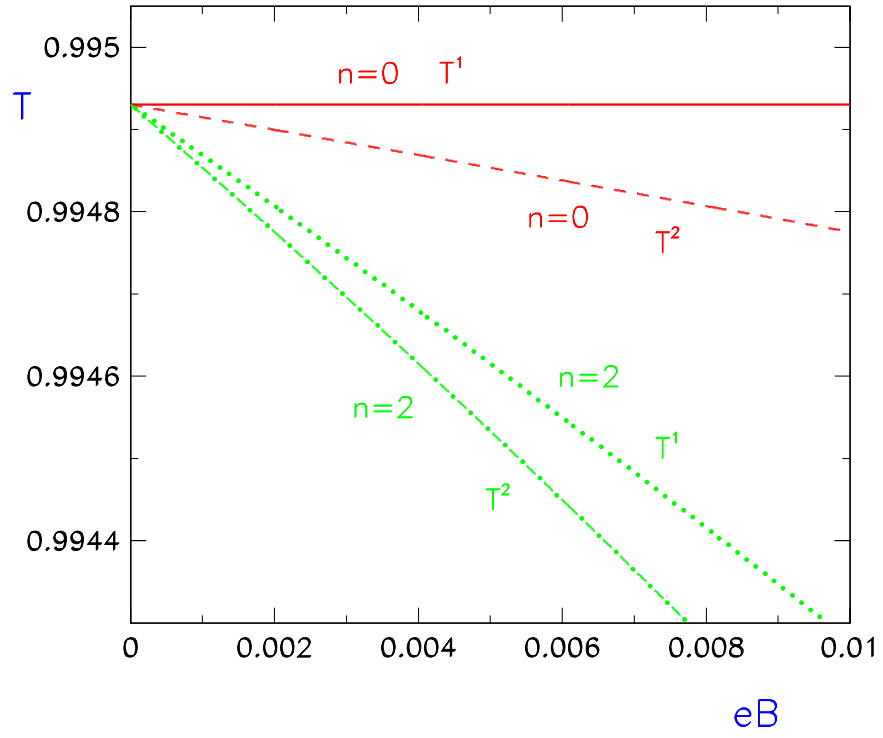
Figure 3: Transmission coefficients for parallel and antiparallel spin at fixed energy as a function of eB for two different values of n.